Int. J. Solids Structures Vol. 17, pp. 267-268
Pergamon Press Ltd., 1981. Printed in Great Britain

0020-7683/81/0201-0267/$02.00/0

COMMENTS ON "ON THE SUBSTANCE OF RIVLIN'S REMARKS ON THE ENDOCHRONIC THEORY", BY K. C. VALANIS

R. S. Rivlin
Lehigh University, Bethlehem, PA 18015, U.S.A.
(Received 24 July 1980)

Readers of Valanis's response[1] to my paper[2] should not be misled by the smokescreen of incorrect statements, irrelevancies and abuse into believing that he has satisfactorily answered any of the substantive criticisms of his theory raised by me.

The main point of my paper is the following: does the constitutive equation advanced by Valanis in his 1971 papers in the *Archive of Mechanic* provide an appropriate basis for describing the mechanical properties of metals? The main objections to Valanis's constitutive equation from this point of view are:

(i) The relation (3.8) in my paper between the incremental moduli for uniaxial loading and unloading, which follows from his theory, is not satisfied by most, and perhaps all, metals. Valanis appears to agree that this is the case.

(ii) When a uniaxial loading history is followed by an infinitesimal cycle of strain, the work done is negative. I claimed in [2] that this implies an instability in the material. Valanis's response to this will be discussed below.

(iii) It is assumed that Poisson's ratio is constant, even when the metal is plastically deformed. This is not the case in practice. Valanis does not address himself to this criticism.

(iv) The constitutive equation of Valanis leads to the conclusion that two strain histories, which are infinitesimally different in the sense of the supremum norm, but lead to the same current value of the strain, may result in quite different values of the stress. This implies that it would be impossible to carry out meaningful stress-strain measurements on a material described by Valanis's constitutive equation. Valanis has responded to this criticism by describing a mechanical system which allegedly shows a similar lack of continuity in the relation between force and displacement. While this may be interesting, if correct, it does not in any way invalidate the criticism.

(v) It was pointed out that the agreement of the constitutive equation of Valanis with the torsion-extension experiments of Mair and Pugh imply only that his constitutive equation is not inconsistent with the experiments. Valanis does not address himself to this criticism. (However, see criticism (iv).)

In discussing objection (ii), Valanis points out, quite correctly, that in linear viscoelastic materials, if we load uniaxially, in some time interval $[0, t]$ and then subject the material to an infinitesimal unloading-loading cycle of uniaxial strain, the work done is negative. He sees no objection to a similar situation existing in a material obeying his constitutive equation. However, there is a very significant difference. In the case of the linear viscoelastic material it can be proven that this does not imply instability, while the converse is the case for a material obeying the Valanis constitutive equation..

To see this we consider the following uniaxial strain history for a linear viscoelastic material. The material is loaded at a constant strain rate K in the time-interval $[0, t]$ and then subjected to N infinitesimal unloading-loading cycles of strain at constant strain rates $-K$ and K respectively. Let the duration of each of these cycles be $(T-t)/N$. We calculate the stress at time T and then take the limit as $N \to \infty$ and the amplitude of the cycles $\to 0$, with T and K fixed. It can easily be shown that the stress σ at time t is given (with the notation of Valanis's eqn (27) in [1]) by

$$\sigma = \frac{EK}{\alpha}\{e^{-\alpha(T-t)} - e^{-\alpha T}\}.$$

This is precisely the same value for σ which is obtained if the material is subjected to uniaxial straining with strain-rate K in the time interval $(0, t]$ and then held at strain Kt until time T.

If the same sequence of monotonic loading and unloading–loading strain cycles is applied to a material satisfying Valanis's constitutive equation the stress at time T is given by

$$\sigma = E_0L_0 + \frac{E_1}{\beta n}\frac{(1+\beta L_0)^n}{(1+\beta L)^{n-1}}\left\{1 - \frac{1}{(1+\beta L_0)^n}\right\}$$

where L_0 and L are the values of l at times t and T respectively. Here the notation of eqns (3.1) and (3.6) in [2] is used. On the other hand if the strain is increased monotonically in the time interval t and then held fixed, the stress at time T will have the higher value

$$\sigma = E_0L_0 + \frac{E_1}{\beta n}(1+\beta L_0)\left\{1 - \frac{1}{(1+\beta L_0)^n}\right\}.$$

The essential point which gives rise to the difference between the viscoelastic and endochronic cases is that when the material is held at fixed deformation, endochronic time remains constant while natural time continues to increase.

Valanis raises a myriad of minor points, which are irrelevant to the main issue. Most of these are, in my opinion, incorrect or misleading. However, they are not of sufficient moment for discussion of them to merit the many pages of this Journal that it would occupy.

REFERENCES

1. K. C. Valanis, On the substance of Rivlin's remarks on the endochronic theory. *Int. J. Solids Structures* **17**, 249–265 (1981).
2. R. S. Rivlin, Some comments on the endochronic theory of plasticity. *Int. J. Solids Structures* **17**, 231–248 (1981).